Detection Strategies for a Multi-Interferometer Triggered Search

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Outline

Background

- review gravitational waves and sources
- review gravitational wave detectors, noise, tests

LR test for burst signal detection

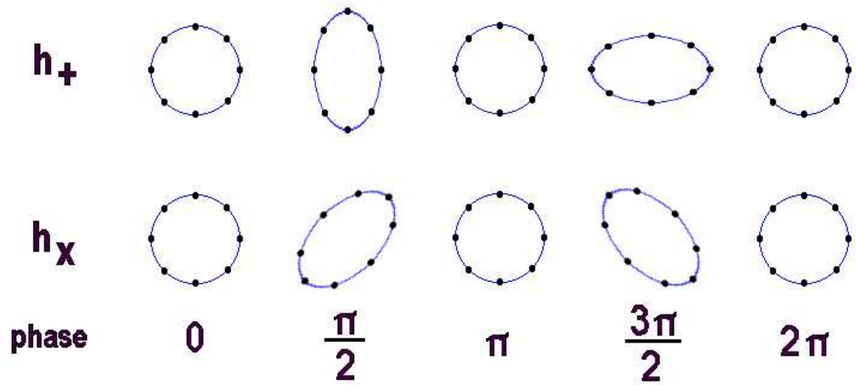
- analytic and simulation comparison of likelihood ratio (LR) and cross-correlation (CC) tests for Gaussian noise
- use simulation to compare LR and CC tests for non-Gaussian noise
- CC test is better for low SNR

General networks of detectors

- want to derive LR statistic, then extract CC part
- LR statistic for two detectors–no CC part
- LR statistic for three or more detectors–expression for CC part
- code to compare CC weightings
- pending issues–polarization angle, correlation matrices

Conclusions and future work

Nature of gravitational waves



Two polarizations of gravitational waves: $+$, $\times$, offset by 45°

Effect is to induce periodic stretching/compression of space-time orthogonal to direction of wave propagation

Gamma ray bursts

First detected in 1967: unexplained short (< 1 s) gamma ray bursts

Compton GRO in 1991: observed 2700 GRBs in nine years, found
- isotropic distribution–not galactic objects
- two classes–short (< 2 s) and long (> 2 s)

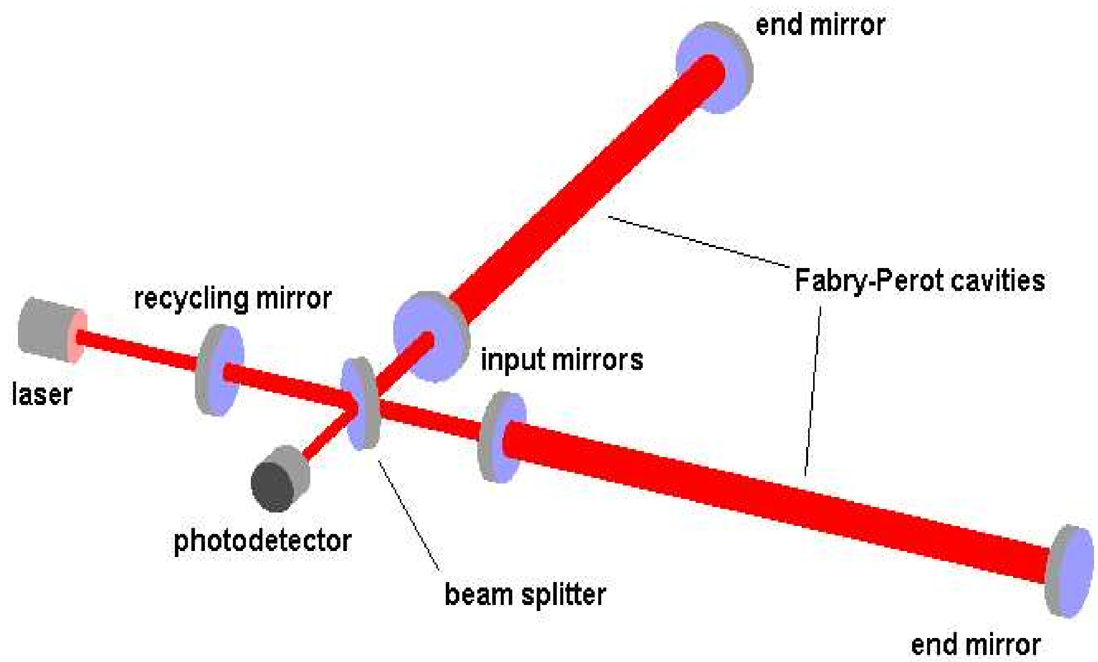
GRB afterglows observed beginning 1997
- identified with host galaxies–cosmological in origin
- redshift measurements give distances–high energies: 10^{54} ergs if anisotropic

Some long GRBs identified with supernovae

Collapsar model for long GRBs
- stellar collapse, core becomes black hole
- infalling material forms accretion disk
- particles/radiation emitted in axial relativistic jets
- jets collide with gas, produce GRBs

Short GRBs still a mystery: no afterglows observed–binary neutron star in-spiral?

Interferometer gravitational wave detectors

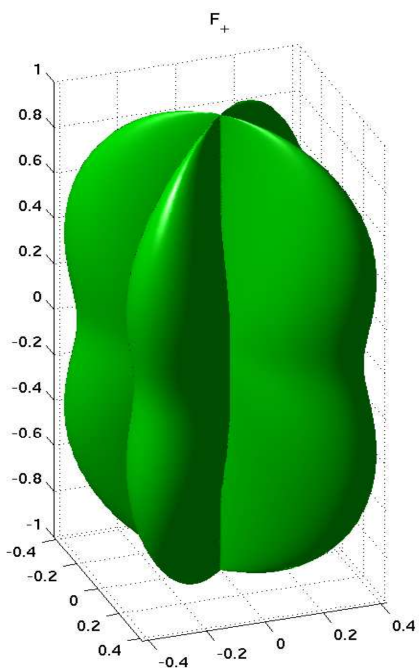


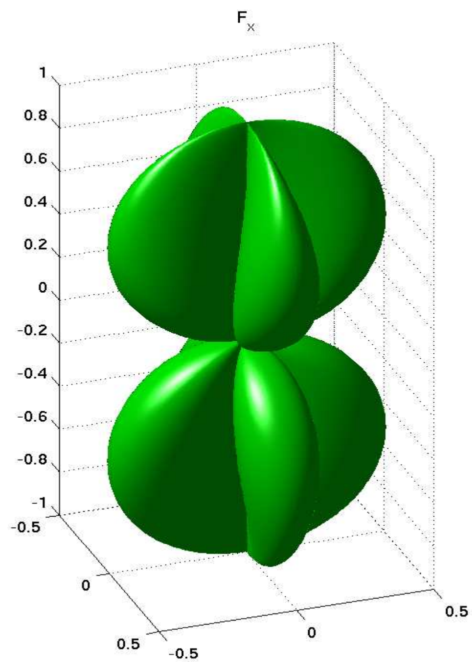
Use a Michelson interferometer as a detector
- passing GW changes relative length of arms
- recombined beam interference compares arm lengths

Existing projects include LIGO (LLO, LHO-2k, LHO-4k), VIRGO, TAMA, GEO

Future space-based project is LISA

Antenna patterns





directional dependence of IFO sensitivity:

$$
\begin{aligned}
F_+(t) &= \sin\xi[a\cos 2\psi + b\sin 2\psi] \\
F_\times(t) &= \sin\xi[b\cos 2\psi - a\sin 2\psi]
\end{aligned}
$$

a, b = functions of source α, δ, detector λ, L, γ, and t

time delay: up to 10 ms between LLO and LHO

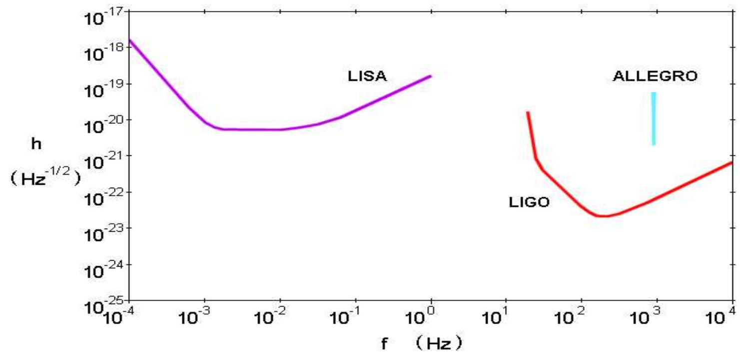
Noise

Ideal noise is Gaussian

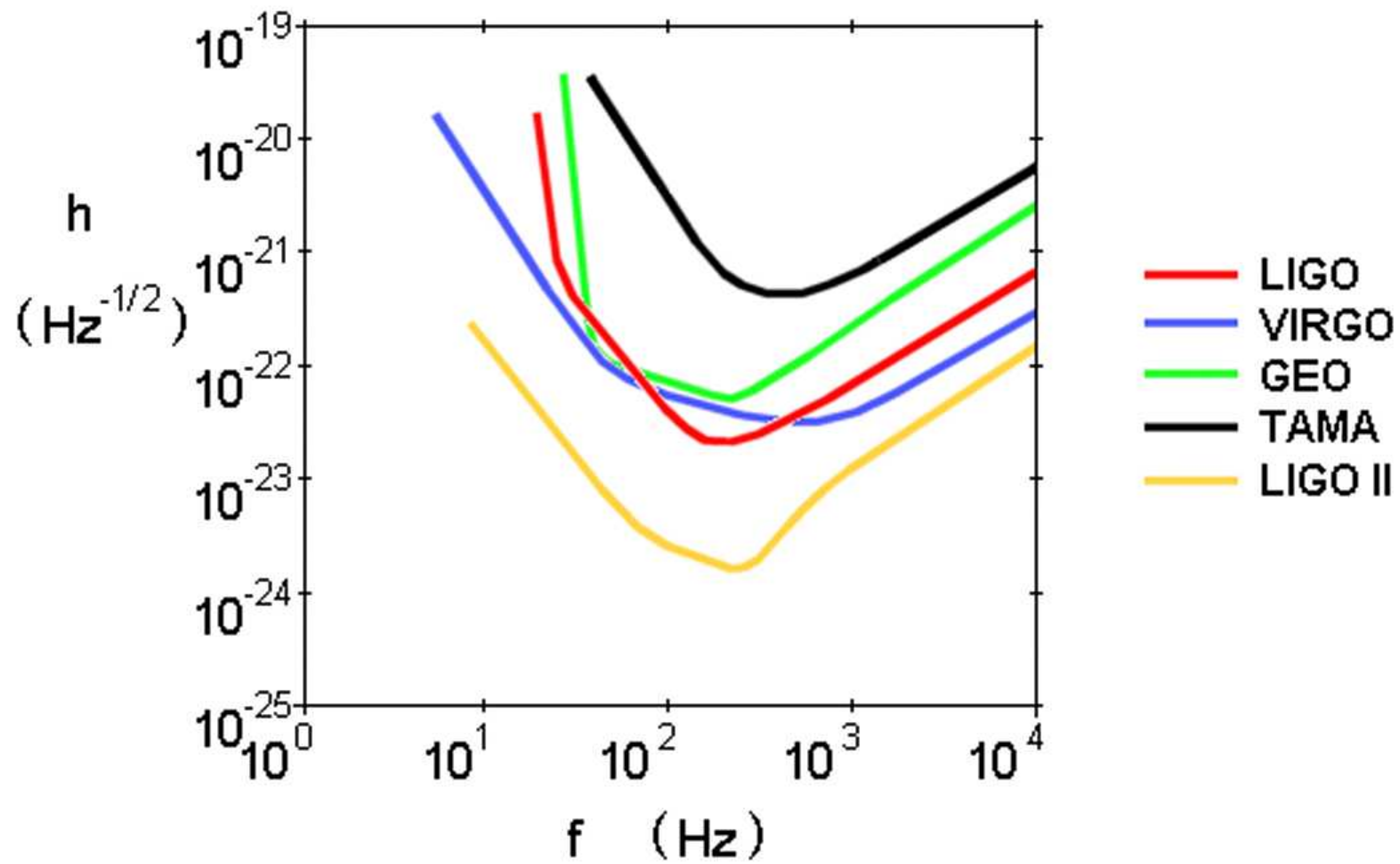
$$f(x(i):\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x(i)-\mu)^2/2\sigma^2} \tag{1}$$

Noise generally non-ideal:
- colored (samples correlated, frequency-dependent sensitivity)
- non-stationary (characteristics vary in time)
- non-Gaussian (e.g. bursts of higher noise)

Covariance of samples (colored noise) can be described in frequency domain by power spectral density



Power spectral densities for interferometers



(shows idealized design sensitivities)

Tests

Receiver operating characteristic (ROC) curve is the relation of the probabilities (P):

$$P\{\Lambda > k \text{ given a signal is present }\} \text{ vs. } P\{\Lambda > k \text{ given no signal }\}$$

for a specified threshold k, where Λ is the test statistic.

Consider time series signal in detector i as $s_i(m) = n_i(m) + h_i(m)$:
$n_i(m)$ is the noise in detector i, hopefully uncorrelated
$h_i(m)$ is the signal

Cross-correlation test:

$$\Lambda_{CC} \equiv \left\langle s_1, s_2 \right\rangle = \sum_{m=1}^{N} s_1(m) s_2(m) \tag{2}$$

Likelihood ratio statistic

If the likelihood ratio test statistic for two co-aligned co-located detectors is generalized to unknown signal $\bar{h}$:

$$\Lambda(\bar{x}|\bar{h}) = \max_{\bar{h}} \frac{P_1(\bar{x}_1, \bar{x}_2|\bar{h})}{P_0(\bar{x}_1, \bar{x}_2)}$$

this statistic can be maximized by maximizing $\ln(\Lambda)$, and

$$\ln(\Lambda) = \sum_{i=1}^{N} x_{1,i} h_i + \sum_{i=1}^{N} x_{2,i} h_i - \sum_{i=1}^{N} h_i^2.$$

This is maximized if

$$\frac{\partial}{\partial h_j} \textstyle\sum_{i=1}^{N} (x_{1,i} h_i + x_{2,i} h_i - h_i^2) = 0 \quad \text{or} \quad h_j = \frac{x_{1,j} + x_{2,j}}{2}.$$

Substituting this result for h_i above gives a maximized statistic

$$\ln(\Lambda) = \left\langle \frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2} \right\rangle$$

which can be compared to a specified theshold k.

LR test vs. CC test for simple pair of detectors

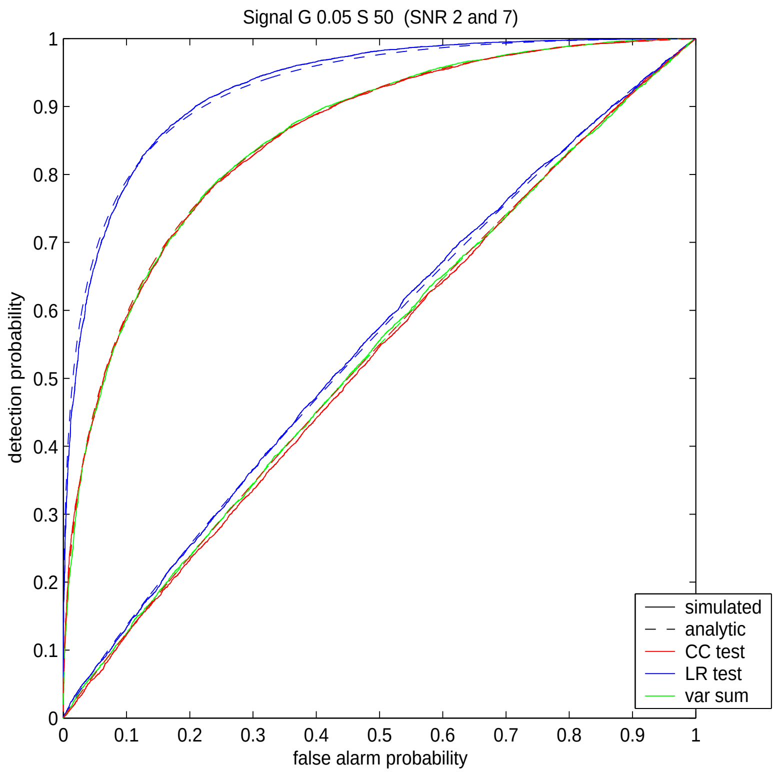
Initially investigated statistics for two identical co-located co-aligned detectors, Gaussian noise

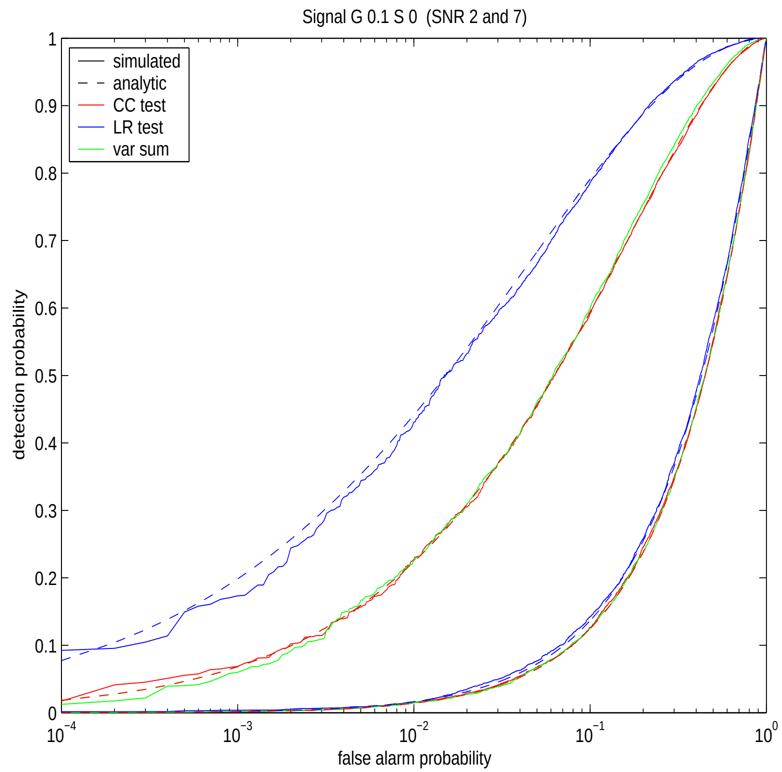
Analytically compared three test statistics:
- likelihood ratio (LR)
- cross-correlation (CC)
- sum of variances (VS) (sum of auto-correlation terms)

Each statistic, for a sufficiently large number of time samples, has a Gaussian distribution.
Mean and variance will be different between cases with signal present and signal absent

Wrote computer code to conduct Monte Carlo simulations:
- For each detector produces a time series as a sum of a specified signal and random noise
- Each trial involves independently doing this for each test, calculating statistics, and storing values
- For specified number of trials (generally 10^3 to 10^4) can count number of cases in which statistics exceed a given threshold, both with and without signal
- These counts provide false alarm probability vs. detection probabilty as threshold is varied.

Simulation results, Gaussian noise





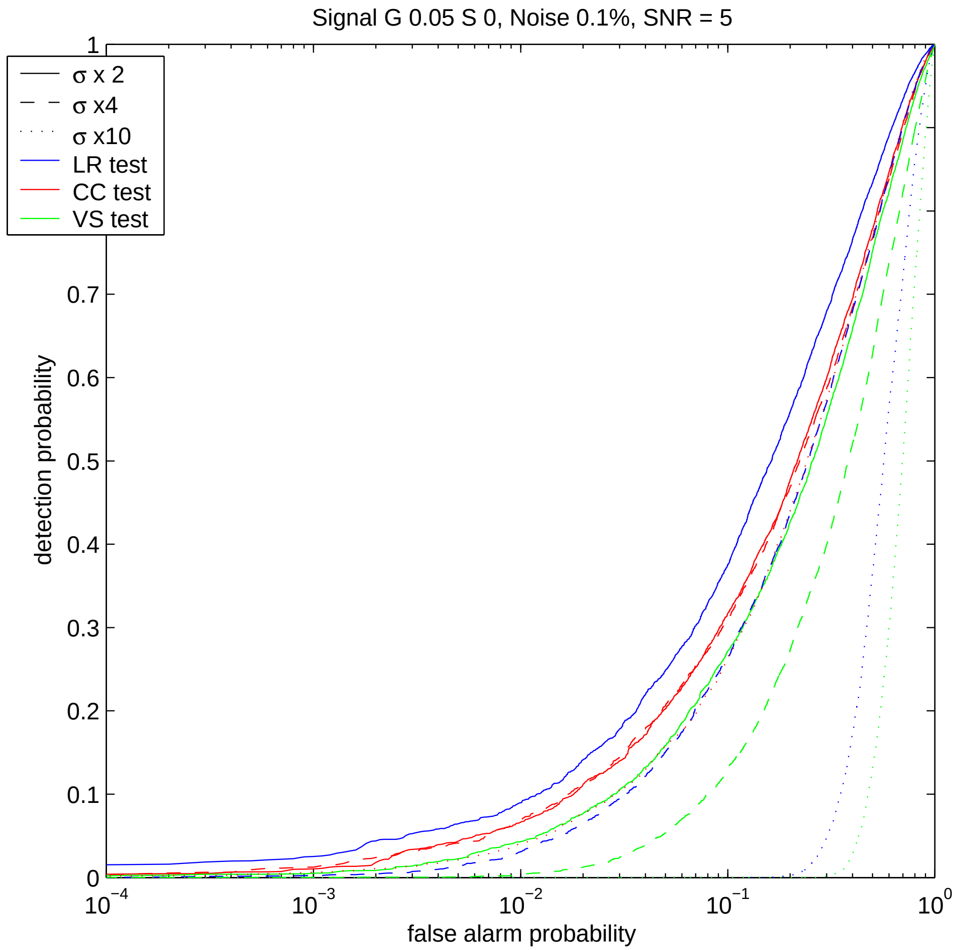
ROC curve for sine-Gaussian signal, 10000 Monte Carlo trials, SNR=2 and SNR=7. Number of samples N=1024.

ROC curve (semi-log) for Gaussian signal, 10000 Monte Carlo trials, SNR=2 and SNR=7. Number of samples N=1024.

Results: LR beats CC test, simulation checks

Simulation, mixed Gaussian

explain noise model picture of results

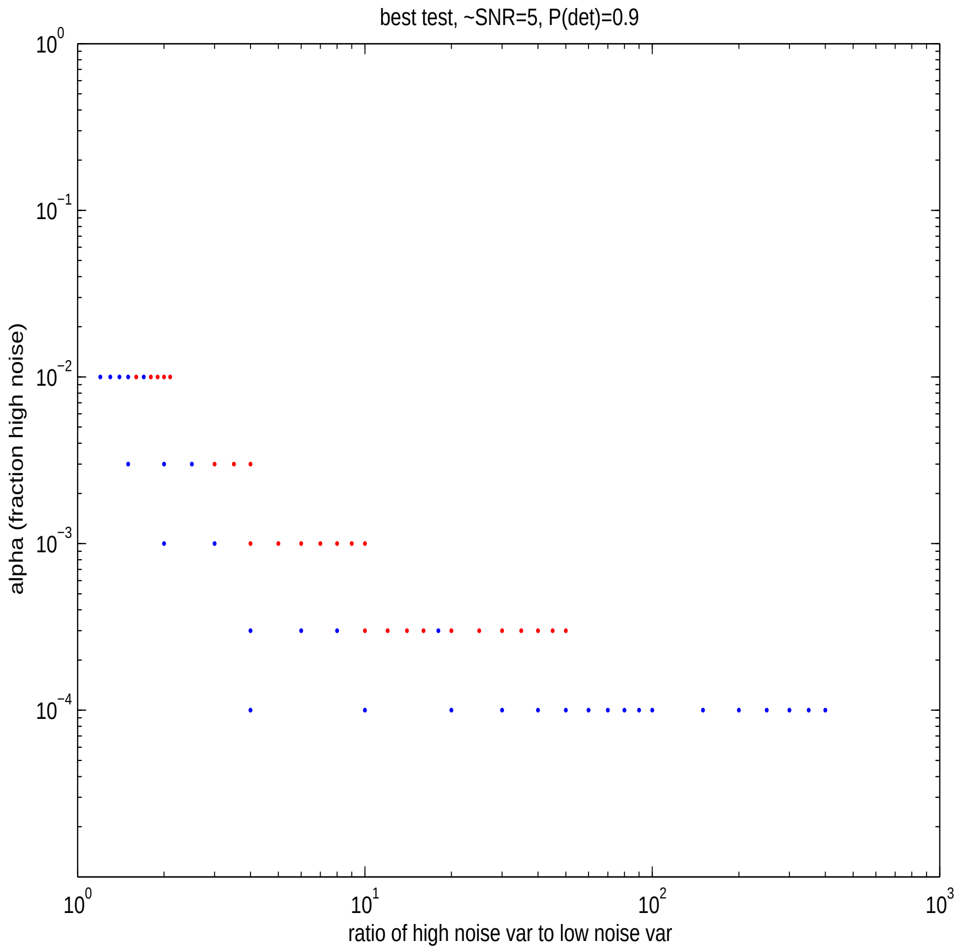


Apply Monte Carlo simulation to more realistic noise

Noise model is mixed Gaussian

Results: cross correlation test performs best for lower SNRs.

Cumulative results, non-Gaussian



Plot of results for various Monte Carlo simulations (each point is run with 10^4 trials)

red = CC test best
blue = LR test best

At low SNR, CC test is best

This applies even for very small fraction of higher noise component

Strategy

Use LR method to derive optimal statistic, keep only CC part

Result is optimal weightings of various CC components

Conditions:

- noise is uncorrelated and white, but may be non-Gaussian
- source location is known from trigger, allowing information on directional response and elimination of time offset
- GW signal waveform is unknown (initially assume known polarization angle)
- initially assume detectors are identical

2-detector network

LR derivation yields

$$\ln(\Lambda) = \sum_{i=1}^{N} \left[\frac{1}{2} x_{1,i}^2 + \frac{1}{2} x_{2,i}^2 \right] \tag{3}$$

(Does not apply to co-located co-aligned detector pair)

Result: no cross-correlation component!

Reason: solution for signal waveform is unconstrained– optimal solution from full space of possible waveforms may be physically unrealistic

Further work could identify a meaningful constraint

3-detector network

$$\ln(\Lambda) = \sum_{i=1}^{N} \left[\kappa_{11} x_{1,i}^2 + \kappa_{22} x_{2,i}^2 + \kappa_{33} x_{3,i}^2 + \kappa_{12} x_{1,i} x_{2,i} + \kappa_{13} x_{1,i} x_{3,i} + \kappa_{23} x_{2,i} x_{3,i} \right] \quad (4)$$

where

$$\kappa_{11} = \frac{1}{2\beta}\left(f_{12}^2 + f_{13}^2\right), \qquad \kappa_{22} = \frac{1}{2\beta}\left(f_{12}^2 + f_{23}^2\right), \quad (5)$$

$$\kappa_{33} = \frac{1}{2\beta}\left(f_{13}^2 + f_{23}^2\right), \qquad \kappa_{12} = \frac{1}{\beta} f_{13} f_{23}, \quad (6)$$

$$\kappa_{13} = -\frac{1}{\beta} f_{12} f_{23}, \qquad \text{and} \qquad \kappa_{23} = \frac{1}{\beta} f_{12} f_{13} \quad (7)$$

using the definitions $f_{pq} = F_{+p} F_{\times q} - F_{+q} F_{\times p}$ and $\beta = f_{12}^2 + f_{13}^2 + f_{23}^2$.

Note that $\kappa_{pp} > 0$, that $\kappa_{11} + \kappa_{22} + \kappa_{33} = 1$, and that κ_{pq} for p not equal to q may be either positive, negative, or zero.

Result: CC terms appear again

Expression weightings for unequal detectors

Previous expression assumed identical detectors.

PSDs are similar in shape for most IFOs, but differ by a scale factor.

Approximation: normalize all detector PSDs to same noise, then incorporate a scale factor g_j for each detector to account for differing response. Then

$$f_{pq} = g_p g_q (F_{+p} F_{\times q} - F_{+q} F_{\times p}) \qquad \text{and} \tag{8}$$

Apply to LIGO: LHO-4k ($g_1 = 1$), LHO-2k ($g_2 = 0.5$), LLO ($g_3 = 1$):

$$\kappa_{11} = \frac{2}{5} = \kappa_{12}, \qquad \kappa_{22} = \frac{1}{10}, \tag{9}$$

$$\kappa_{33} = \frac{1}{2}, \qquad \text{and} \qquad \kappa_{13} = \kappa_{23} = 0. \tag{10}$$

n-detector network

$$\ln(\Lambda) = \sum_{i=1}^{N}\left[\sum_{j=1}^{n}\kappa_{jj}x_{j,i}^{2} + \sum_{j=1}^{n}\sum_{k=1}^{j-1}\kappa_{jk}x_{j,i}x_{k,i}\right] \tag{11}$$

where

$$\kappa_{jj} = \frac{1}{2\beta}\sum_{k\neq j}^{n} f_{jk}^{2} \qquad \kappa_{jk} = \frac{1}{\beta}\sum_{p\neq j,k}^{n} f_{jp}f_{kp} \tag{12}$$

$$\beta = \sum_{j=1}^{n}\sum_{k=1}^{j-1} f_{jk}^{2} \qquad \text{and} \qquad f_{pq} = g_p g_q (F_{+p}F_{\times q} - F_{+q}F_{\times p}) \tag{13}$$

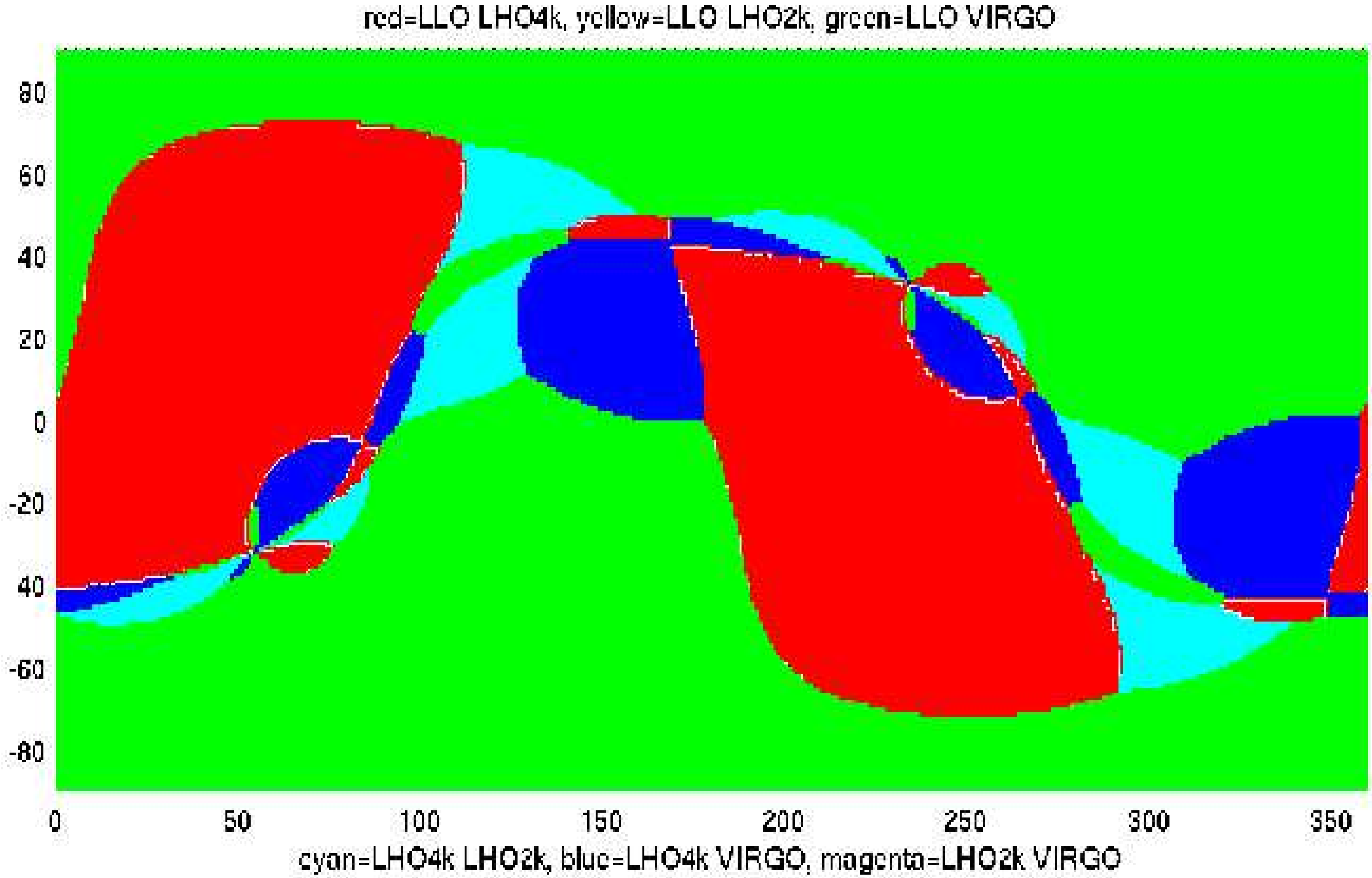
Code to calculate weightings

have expression for weightings as function of

- source location
- detector locations and characteristics
- time

Code gets weightings, makes sky map of greatest mag. component

Illustrative sky map of greatest CC weighting



Map shows which cross-correlation term has the greatest magnitude as a function of sky position. Network includes LLO, both LHO, and VIRGO

Conclusions and future work

Simulation for two co-aligned co-located detectors:
- For non-Gaussian noise and low SNR, CC is better than LR

Used LR method to obtain relative weightings of terms in order to keep CC terms only:
- LR method gives no CC term for two-detector network
- Need to identify additional constraint to apply in LR method

Proceeded with above method for larger networks:
- For larger networks CC terms appear
- Can apply assumption that PSDs are same shape with scaled sensitivity to approximate differing IFO responses
- Developed code which compares weightings of various terms
- Need to investigate incorporating unknown polarization angle
- Need to incorporate varying PSDs (covariance matrices)

Potential to produce improved objective criteria for weighing triggered burst cross-correlations in a multi-detector network.