Detection Strategies for a Multi-Interferometer Triggered Search

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Abstract

This project will compare various statistical tests for signal detection with the objective of identifying optimum tests for signal detection for the LSC triggered burst source analysis. Here, three tests–cross correlation, likelihood ratio, and sum of variances–are compared for two co-aligned co-located detectors. This comparison is done both analytically and with Monte Carlo simulations for the case of uncorrelated Gaussian white noise. These results are consistent, affirming the Monte Carlo results and showing the likelihood ratio test to be consistently superior in performance for this case. The Monte Carlo simulations are then applied to a case of more realistic noise, specifically mixed Gaussian noise. In some such cases examined, the cross correlation test significantly outperforms the likelihood ratio test for low signal-to-noise ratios even if "noisy" samples comprise less than one percent of the time series. Pending work will generalize these tests to greater numbers of detectors and to detectors with different orientations and locations.

Background

The search for gravitational waves using detectors such as LIGO involves low signal-to-noise ratios (SNRs). Statistical tests are a necessary means of positively detecting a signal.

The goal of any test is to maximize detection probability and minimize false alarm probability. These aspects are often illustrated by the receiver operating characteristic (ROC) curve, which is the relation of the probabilities (P):

$$P\{\Lambda > k \text{ given a signal is present }\} \text{ vs. } P\{\Lambda > k \text{ given no signal }\}$$

for a specified threshold k, where Λ is the test statistic.

The signal in detector $i = 1, 2$ is described as $s_i(m) = n_i(m) + h(m)$ for the mth sample of a time series of N samples, where n_i is the noise in detector i and h is the signal (here assumed to be the same for both detectors). Noise is assumed Gaussian (with $\mu = 0$ and $\sigma^2 = 1$) and uncorrelated.

Likelihood ratio statistic

If the likelihood ratio test statistic for two detectors is generalized to unknown signal $\bar{h}$:

$$\Lambda(\bar{x}|\bar{h}) = \max_{\bar{h}} \frac{P_1(\bar{x}_1, \bar{x}_2|\bar{h})}{P_0(\bar{x}_1, \bar{x}_2)}$$

this statistic can be maximized by maximizing $\ln(\Lambda)$, and

$$\ln(\Lambda) = \sum_{i=1}^{N} x_{1,i} h_i + \sum_{i=1}^{N} x_{2,i} h_i - \sum_{i=1}^{N} h_i^2.$$

This is maximized if

$$\frac{\partial}{\partial h_j} \sum_{i=1}^{N} (x_{1,i} h_i + x_{2,i} h_i - h_i^2) = 0 \quad \text{or} \quad h_j = \frac{x_{1,j} + x_{2,j}}{2}.$$

Substituting this result for h_i above gives a maximized statistic

$$\ln(\Lambda) = \left\langle \frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2} \right\rangle$$

which can be compared to a specified theshold k.

Definitions of statistics used

Cross correlation statistic:

$$\Lambda_{CC} \equiv \langle s_1, s_2 \rangle = \sum_{m=1}^{N} s_1(m) s_2(m) \tag{1}$$

Likelihood ratio test statistic:

$$\Lambda_{LR} \equiv \langle \frac{s_1 + s_2}{2}, \frac{s_1 + s_2}{2} \rangle = \frac{1}{4}\langle s_1, s_1 \rangle + \frac{1}{4}\langle s_2, s_2 \rangle + \frac{1}{2}\langle s_1, s_2 \rangle \tag{2}$$

Variance sum test statistic:

$$\Lambda_{VS} \equiv \langle s_1, s_1 \rangle + \langle s_2, s_2 \rangle \tag{3}$$

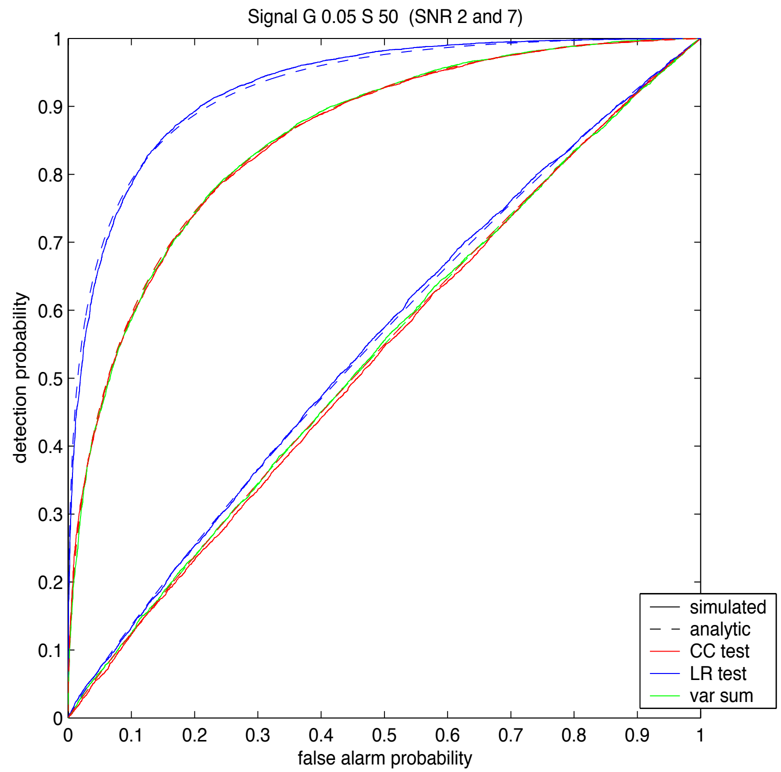
Analytic results

By the central limit theorem, for sufficiently large number of samples N the statistics are Gaussian random variables with the following parameters:

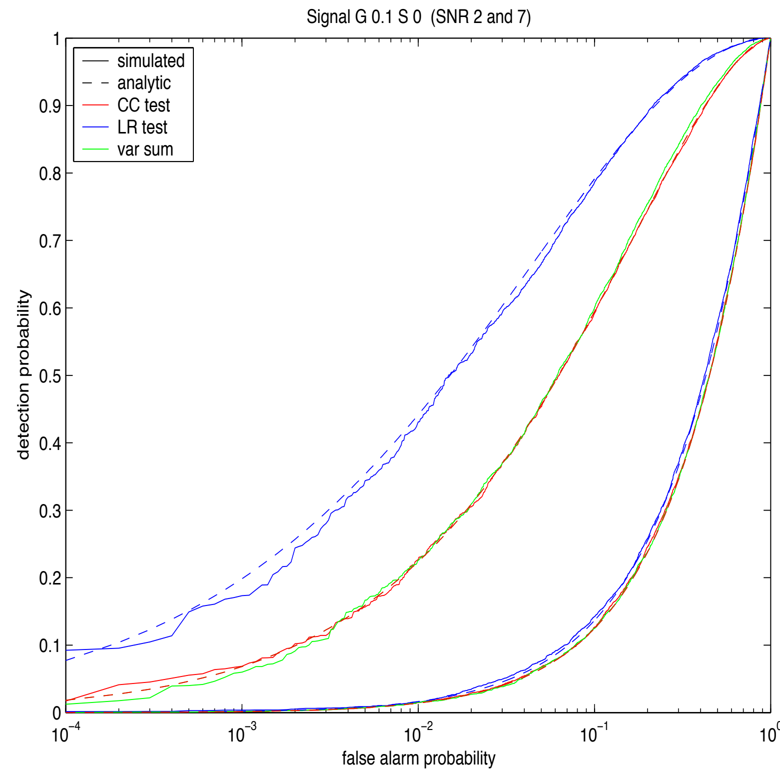
test	signal absent	signal present
cross correlation	$\mu = 0, \sigma^2 = N$	$\mu = C, \sigma^2 = N + 2C$
likelihood ratio	$\mu = \frac{1}{2}N, \sigma^2 = \frac{1}{2}N$	$\mu = \frac{1}{2}N + C, \sigma^2 = \frac{1}{2}N + 2C$
variance sum	$\mu = 2N, \sigma^2 = 4N$	$\mu = 2N + 2C, \sigma^2 = 4N + 8C$

where N is the number of samples and $C = \sum_{m=1}^{N} h^2(m)$, a constant for a given signal.

Comparison of analytic/simulation results for Gaussian noise

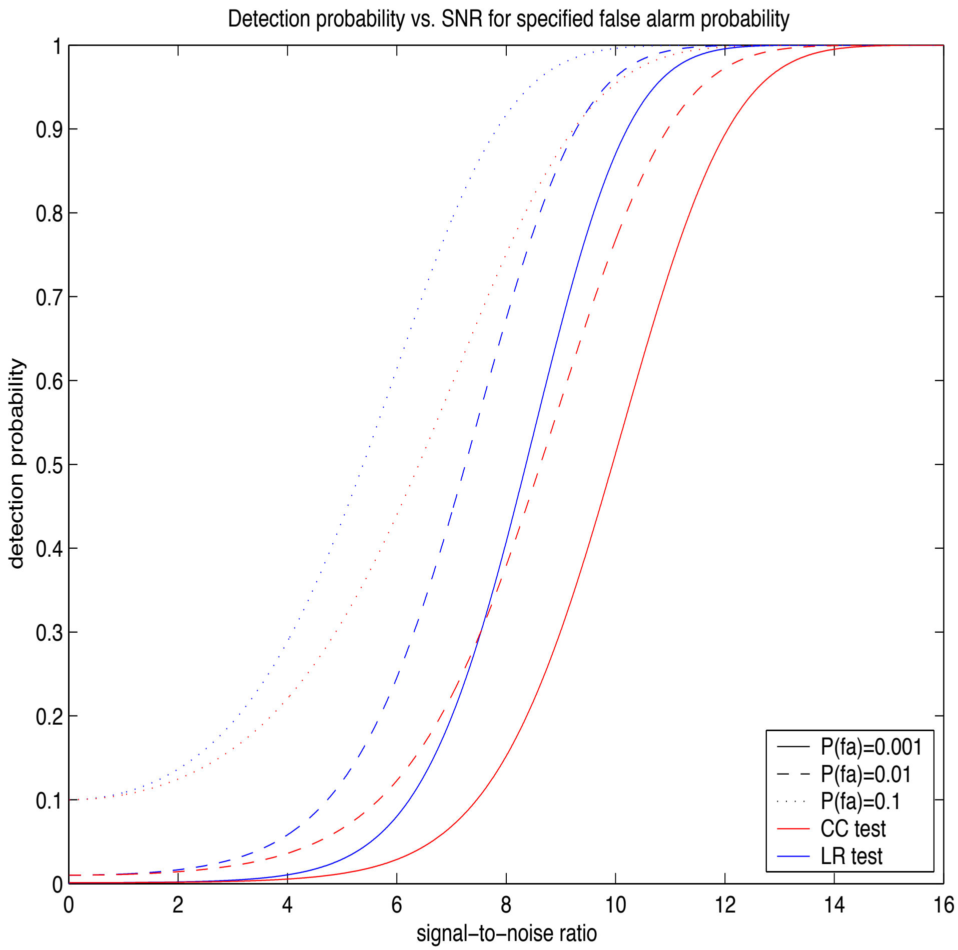


ROC curve for sine-Gaussian signal, 10000 Monte Carlo trials, SNR=2 and SNR=7. Number of samples N=1024.



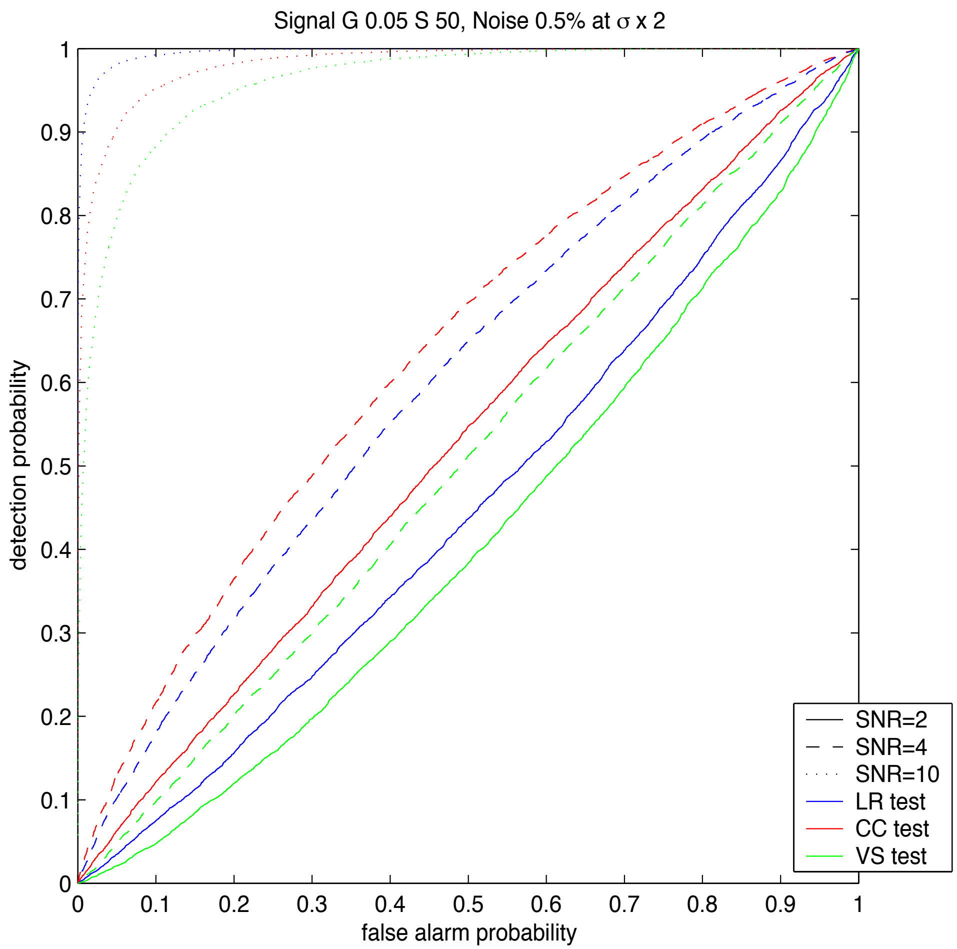
ROC curve (semi-log) for Gaussian signal, 10000 Monte Carlo trials, SNR=2 and SNR=7. Number of samples N=1024.

Analytic results



Analytic results for detection probability vs. SNR, given a fixed false alarm probability P(fa). Results for the variance sum test are indistinguishable from the cross correlation test.

Monte Carlo simulation results for mixed Gaussian noise



Results for simulations (10000 Monte Carlo trials each) with 0.5 percent noisy samples—samples with (in this case) twice the standard deviation as quiet samples. Each series is N=1024 total samples. Note that the cross correlation test performs best for lower SNRs.

Conclusions

For Gaussian noise, analytic results affirm the results of Monte Carlo simulations. The analytic results indicate that the likelihood ratio test is superior to the cross correlation test, and that the relative performance of these tests is independent of the form of the signal beyond dependence on the SNR.

Applied to mixed Gaussian noise, the Monte Carlo simulation shows that at low SNR the cross correlation test is superior in performance, even if the noisy samples comprise less than one percent of the time series. Further work will define the regime of SNR and noise component where this reversal of relative performance occurs.

Forthcoming work will generalize the analytic results to more than two detectors and to multiple detectors at different locations and orientations, appropriate for correlations between LIGO and other IFOs. These results will be in the context of a triggered burst search, i.e. where source direction is known.